Useful Inequalities $\{x^2 \geqslant 0\}$

`version 0.10a · August 1, 2011`

Cauchy-Schwarz $$\left(\sum_{i=1}^{n} x_i y_i\right)^2 \leq \left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} y_i^2\right)$$

Minkowski $$\left(\sum_{i=1}^{n} |x_i + y_i|^p\right)^{\frac{1}{p}} \leq \left(\sum_{i=1}^{n} |x_i|^p\right)^{\frac{1}{p}} + \left(\sum_{i=1}^{n} |y_i|^p\right)^{\frac{1}{p}}, \quad \text{for } p \geq 1.$$

Hölder $$\sum_{i=1}^{n} |x_i y_i| \leq \left(\sum_{i=1}^{n} |x_i|^p\right)^{1/p} \left(\sum_{i=1}^{n} |y_i|^q\right)^{1/q}, \quad \text{for } p, q > 1,\ \frac{1}{p} + \frac{1}{q} = 1.$$

Bernoulli $(1+x)^r \geq 1 + rx$, for $x > -1$, $r \in \mathbb{R} \setminus (0,1)$. If $r = 2n$ $(n \in \mathbb{N})$, inequality holds for $x \in \mathbb{R}$. Reverse holds for $x > -1$, $r \in [0,1]$.

$$(1+x)^r \leq 1 + \frac{rx}{1-(r-1)x}, \quad \text{for } r > 1, \quad -1 \leq x < \frac{1}{r-1}.$$

exponential $e^x \geq \left(1 + \frac{x}{n}\right)^n \geq 1 + x$, for $x \in \mathbb{R}$, $n > 0$ (left), $n \geq 1$ (right).
If $x < 0$, then $n \geq -x$ is required for both. Outer inequality always holds.

$$e^x \geq x^e, \quad \text{for } x \in \mathbb{R}, \qquad e^x \geq 1 + x + \frac{x^2}{2}, \quad \text{for } x \geq 0, \text{ reverse for } x \leq 0.$$

logarithm $$\frac{x}{x+1} \leq \log(1+x) \leq \min\left\{x,\ x - \frac{x^2}{2} + \frac{x^3}{3}\right\}, \quad \text{for } x > -1.$$

$$\frac{2x}{2+x} \leq \log(1+x) \leq \frac{x}{\sqrt{x+1}}, \quad \text{for } x \geq 0. \text{ Reverse for } x \in (-1, 0].$$

$$\log(1+x) \geq x - \frac{x^2}{2} + \frac{x^3}{4}, \quad \text{for } x \in [0, \sim 0.45], \text{ reverse elsewhere.}$$

$$\log(1-x) \geq -x - \frac{x^2}{2} - \frac{x^3}{2}, \quad \text{for } x \in [0, \sim 0.43], \text{ reverse elsewhere.}$$

harmonic $$\log(n+1) \leq \sum_{i=1}^{n} \frac{1}{i} \leq \log(n) + 1$$

square root $$2\sqrt{x+1} - 2\sqrt{x} < \frac{1}{\sqrt{x}} < 2\sqrt{x} - 2\sqrt{x-1}, \quad \text{for } x \geq 1.$$

binomial $$\left(\frac{n}{k}\right)^k \leq \binom{n}{k} \leq \frac{n^k}{k!} \leq \left(\frac{en}{k}\right)^k, \quad \text{for } n \geq k > 0.$$

binomial sum $$\sum_{i=0}^{d} \binom{n}{i} \leq n^d + 1, \quad \text{for } n \geq d \geq 0,$$

$$\sum_{i=0}^{d} \binom{n}{i} \leq \left(\frac{en}{d}\right)^d, \quad \text{for } n \geq d \geq 1.$$

middle binomial $$\frac{2^{2n}}{2\sqrt{n}} \leq \binom{2n}{n} \leq \frac{2^{2n}}{\sqrt{2n}}$$

binomial ratio $$\binom{n}{\alpha n} \leq \left[\alpha^{\alpha}(1-\alpha)^{(1-\alpha)}\right]^{-n}, \quad \text{for } \alpha \in (0,1).$$

Stirling $$e\left(\frac{n}{e}\right)^n \leq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/(12n+1)} \leq n! \leq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{1/12n} \leq en\left(\frac{n}{e}\right)^n$$

trigonometric $$x - \frac{x^3}{2} \leq x\cos x \leq \frac{x\cos x}{1 - x^2/3} \leq x\sqrt[3]{\cos x} \leq x - \frac{x^3}{6} \leq x\cos\frac{x}{\sqrt{3}} \leq \sin x \leq$$
$$x\cos(0.56\,x) \leq x \leq x + \frac{x^3}{3} \leq \tan x, \quad \text{and} \quad \frac{2}{\pi}x \leq \sin x, \quad \text{for } x \in \left[0, \tfrac{\pi}{2}\right].$$

hyperbolic $$\cosh(x) + \alpha\sinh(x) \leq e^{x^2/2 + \alpha x}, \quad \text{where } x \in \mathbb{R},\ \alpha \in [-1, 1].$$

Napier $$b > \frac{a+b}{2} > \frac{b-a}{\ln(b) - \ln(a)} > \sqrt{ab} > a, \quad \text{for } 0 < a < b.$$

means $$\max\{x_i\} \geq \sqrt{\frac{\sum x_i{}^2}{n}} \geq \frac{\sum x_i}{n} \geq \left(\prod x_i\right)^{1/n} \geq \frac{n}{\sum x_i^{-1}} \geq \min\{x_i\}$$

power means $M_w^r \leq M_w^s$, for all pairs $r \leq s$, where:

$$M_w^r(x_1, x_2, \ldots, x_n) = \left(\sum_{i=1}^{n} w_i x_i^r\right)^{1/r} \text{ and } \sum w_i = 1.$$

If $r = -\infty, 0, +\infty$, M_w^r tends to min, geom. mean and max, respectively.

Maclaurin $\sqrt[k]{S_k} \geq \sqrt[(k+1)]{S_{k+1}}$, for $1 \leq k < n$, where:

$$S_k = \frac{1}{\binom{n}{k}} \sum_{1 \leq i_1 < \cdots < i_k \leq n} a_{i_1} a_{i_2} \cdots a_{i_k}, \quad \text{and} \quad a_i > 0.$$

Newton $S_k{}^2 \geq S_{k-1} S_{k+1}$, for $1 \leq k < n$, and S_k as before.

Jensen $\varphi\left(\mathrm{E}[X]\right) \leq \mathrm{E}\left[\varphi(X)\right]$, where X is a random variable, and φ **convex**.
For concave φ the reverse holds. Without probabilities:

$$\varphi\left(\sum_{i=1}^{n} p_i x_i\right) \leq \sum_{i=1}^{n} p_i \varphi(x_i), \quad \text{where } p_i \geq 0,\ \sum p_i = 1.$$

Chebyshev $$\sum_{i=1}^{n} f(a_i) g(b_i) p_i \geq \left(\sum_{i=1}^{n} f(a_i) p_i\right)\left(\sum_{i=1}^{n} g(b_i) p_i\right) \geq \sum_{i=1}^{n} f(a_i) g(b_{n-i+1}) p_i,$$

for $a_1 \leq \cdots \leq a_n$, $b_1 \leq \cdots \leq b_n$, and f, g nondecreasing, $p_i \geq 0$, $\sum p_i = 1$.
With expectations: $\mathrm{E}\big[f(X)g(X)\big] \geq \mathrm{E}\big[f(X)\big]\mathrm{E}\big[g(X)\big]$.

rearrangement $$\sum_{i=1}^{n} a_i b_i \geq \sum_{i=1}^{n} a_i b_{\pi(i)} \geq \sum_{i=1}^{n} a_i b_{n-i+1}, \quad \text{for } a_1 \leq \cdots \leq a_n,$$

$b_1 \leq \cdots \leq b_n$ and π a permutation of $[n]$. More generally:

$$\sum_{i=1}^{n} f_i(b_i) \geq \sum_{i=1}^{n} f_i(b_{\pi(i)}) \geq \sum_{i=1}^{n} f_i(b_{n-i+1}),$$

with $\big(f_{i+1}(x) - f_i(x)\big)$ nondecreasing for all $1 \leq i < n$.

Young $$xy \leq \frac{x^p}{p} + \frac{y^q}{q}, \quad \text{for } x, y \geq 0 \text{ and } p, q > 0,\ \frac{1}{p} + \frac{1}{q} = 1.$$

Chong $$\sum_{i=1}^{n} \frac{a_i}{a_{\pi(i)}} \geq n, \quad \text{and} \quad \prod_{i=1}^{n} a_i{}^{a_i} \geq \prod_{i=1}^{n} a_i{}^{a_{\pi(i)}}, \quad \text{for } a_i > 0.$$

Kantorovich $\left(\sum_{i=1}^n {x_i}^2\right)\left(\sum_{i=1}^n {y_i}^2\right) \le \left(\frac{A}{G}\right)^2 \left(\sum_{i=1}^n x_i y_i\right)^2$, for $x_i, y_i > 0$,
$0 < m \le \frac{x_i}{y_i} \le M < \infty$, $A = (m+M)/2$, $G = \sqrt{mM}$.

Cauchy $\varphi'(a) \le \frac{f(b)-f(a)}{b-a} \le \varphi'(b)$, where $a < b$, and φ convex.
For concave φ the reverse holds.

Hadamard $\varphi\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b \varphi(x)\,dx \le \frac{\varphi(a)+\varphi(b)}{2}$, for φ convex.

Gibbs $\sum_{i=1}^n a_i \log \frac{a_i}{b_i} \ge a \log \frac{a}{b}$, for $a_i, b_i \ge 0$, $a := \sum a_i$, $b := \sum b_i$.

Woeginger $\sum_{i=1}^n a_i \varphi(\frac{b_i}{a_i}) \le a\,\varphi(\frac{b}{a})$, for φ **concave** and variables as before.

Pečarić $(1+\frac{x}{p})^p \ge (1+\frac{x}{q})^q$, where either (i) $x > 0$, $p > q > 0$,
(ii) $-p < -q < x < 0$ or (iii) $-q > -p > x > 0$. Reverse, if
(iv) $q < 0 < p$, $-q > x > 0$ or (v) $q < 0 < p$, $-p < x < 0$.

Shapiro $\sum_{i=1}^n \frac{x_i}{x_{i+1}+x_{i+2}} \ge \frac{n}{2}$, where $x_i > 0$, $(x_{n+1}, x_{n+2}) := (x_1, x_2)$,
and $n \le 12$ if even, $n \le 23$ if odd.

Schur $x^t(x-y)(x-z) + y^t(y-z)(y-x) + z^t(z-x)(z-y) \ge 0$,
where $x, y, z \ge 0$, $t > 0$

Weierstrass $\prod_{i=1}^n (1-x_i)^{w_i} \ge 1 - \sum_{i=1}^n w_i x_i$, where $x_i \le 1$, and
either $w_i \ge 1$ (for all i) or $w_i \le 0$ (for all i).
If $w_i \in [0,1]$, $\sum w_i \le 1$, and $x_i \le 1$, the reverse holds.

Ky Fan $\frac{\prod_{i=1}^n x_i^{a_i}}{\prod_{i=1}^n (1-x_i)^{a_i}} \le \frac{\sum_{i=1}^n a_i x_i}{\sum_{i=1}^n a_i(1-x_i)}$, for $x_i \in [0, \frac{1}{2}]$, $a_i \in [0,1]$, $\sum a_i = 1$.

Aczél $(a_1 b_1 - \sum_{i=2}^n a_i b_i)^2 \ge (a_1^2 - \sum_{i=2}^n a_i^2)(b_1^2 - \sum_{i=2}^n b_i^2)$,
given that $a_1^2 > \sum_{i=2}^n a_i^2$ or $b_1^2 > \sum_{i=2}^n b_i^2$.

Callebaut $\left(\sum_{i=1}^n a_i^{1+x} b_i^{1-x}\right)\left(\sum_{i=1}^n a_i^{1-x} b_i^{1+x}\right) \ge \left(\sum_{i=1}^n a_i^{1+y} b_i^{1-y}\right)\left(\sum_{i=1}^n a_i^{1-y} b_i^{1+y}\right)$,
for $1 \ge x \ge y \ge 0$.

Mahler $\prod_{i=1}^n (x_i+y_i)^{1/n} \ge \prod_{i=1}^n x_i^{1/n} + \prod_{i=1}^n y_i^{1/n}$, where $x_i, y_i > 0$.

unknown $\sum_{j=1}^m \prod_{i=1}^n a_{ij} \ge \sum_{j=1}^m \prod_{i=1}^n a_{i\pi(j)}$, and $\prod_{j=1}^m \sum_{i=1}^n a_{ij} \le \prod_{j=1}^m \sum_{i=1}^n a_{i\pi(j)}$,
for $0 \le a_{i1} \le \cdots \le a_{im}$ for $i = 1, \ldots, n$ and π is a permutation of $[n]$.

Karamata $\sum_{i=1}^n \varphi(a_i) \ge \sum_{i=1}^n \varphi(b_i)$, where $a_1 \ge a_2 \ge \cdots \ge a_n$ and $b_1 \ge \cdots \ge b_n$,
and $\{a_i\} \succeq \{b_i\}$ (majorization), i.e. $\sum_{i=1}^t a_i \ge \sum_{i=1}^t b_i$ for all $1 \le t \le n$,
with equality for $t = n$ and φ is convex (for concave φ the reverse holds).

Muirhead $\frac{1}{n!}\sum_\pi x_{\pi(1)}^{a_1} \cdots x_{\pi(n)}^{a_n} \ge \frac{1}{n!}\sum_\pi x_{\pi(1)}^{b_1} \cdots x_{\pi(n)}^{b_n}$,
where $a_1 \ge a_2 \ge \cdots \ge a_n$ and $b_1 \ge b_2 \ge \cdots \ge b_n$ and $\{a_k\} \succeq \{b_k\}$,
$x_i \ge 0$ and the sums extend over all permutations π of $[n]$.

Carleman $\sum_{k=1}^n \left(\prod_{i=1}^k |a_i|\right)^{1/k} \le e \sum_{k=1}^n |a_k|$

Milne $\left(\sum_{i=1}^n (a_i+b_i)\right)\left(\sum_{i=1}^n \frac{a_i b_i}{a_i+b_i}\right) \le \left(\sum_{i=1}^n a_i\right)\left(\sum_{i=1}^n b_i\right)$

Abel $b_n \min_k \sum_{i=1}^k |a_i| \le \sum_{i=1}^n |a_i b_i| \le b_n \max_k \sum_{i=1}^k |a_i|$, for $0 \le b_1 \le \cdots \le b_n$.

Hilbert $\sum_{m=1}^\infty \sum_{n=1}^\infty \frac{a_m b_n}{m+n} \le \pi \left(\sum_{m=1}^\infty a_m^2\right)^{\frac{1}{2}} \left(\sum_{n=1}^\infty b_n^2\right)^{\frac{1}{2}}$, for $a_m, b_n \in \mathbb{R}$.
If we put $\max\{m,n\}$ instead of $m+n$, we have 4 instead of π.

Hardy $\sum_{n=1}^\infty \left(\frac{a_1+a_2+\cdots+a_n}{n}\right)^p \le \left(\frac{p}{p-1}\right)^p \sum_{n=1}^\infty a_n^p$, for $a_n \ge 0$, $p > 1$.

Carlson $\left(\sum_{n=1}^\infty a_n\right)^4 \le \pi^2 \sum_{n=1}^\infty {a_n}^2 \sum_{n=1}^\infty n^2 {a_n}^2$, for $a_n \in \mathbb{R}$.

Mathieu $\frac{1}{c^2+1/2} < \sum_{n=1}^\infty \frac{2n}{(n^2+c^2)^2} < \frac{1}{c^2}$, for $c \ne 0$.

Copson $\sum_{n=1}^\infty \left(\sum_{k \ge n} \frac{a_k}{k}\right)^p \le p^p \sum_{n=1}^\infty {a_n}^p$, for $a_n \ge 0$, $p > 1$, reverse if $p \in (0,1)$.

Bonferroni

$$\Pr\left[\bigcup_{i=1}^{n} A_i\right] \le \sum_{j=1}^{k} (-1)^{j-1} S_j, \quad \text{for } 1 \le k \le n,\ k \text{ odd},$$

$$\Pr\left[\bigcup_{i=1}^{n} A_i\right] \ge \sum_{j=1}^{k} (-1)^{j-1} S_j, \quad \text{for } 2 \le k \le n,\ k \text{ even}.$$

$$S_k = \sum_{1 \le i_1 < \cdots < i_k \le n} \Pr\big[A_{i_1} \cap \cdots \cap A_{i_k}\big], \quad \text{where } A_i \text{ are events.}$$

Markov

$$\Pr\big[|X| \ge a\,\big] \le \frac{\mathrm{E}\big[|X|\big]}{a}, \quad \text{where } X \text{ is a random variable, } a > 0.$$

$$\Pr\big[X \le c\,\big] \le \frac{1 - \mathrm{E}[X]}{1 - c}, \quad \text{for } X \in [0,1] \text{ and } c \in \big[0, \mathrm{E}[X]\big].$$

Without probabilities: $c \le \frac{n\mu}{a}$, where c is the number of elements $\ge a$, among n nonnegative numbers with mean μ.

Chebyshev

$$\Pr\big[\,\big|X - \mathrm{E}[X]\big| \ge t\,\big] \le \frac{\mathrm{Var}[X]}{t^2},$$

$$\Pr\big[X - \mathrm{E}[X] \ge t\,\big] \le \frac{\mathrm{Var}[X]}{\mathrm{Var}[X] + t^2}, \quad \text{where } t > 0 \text{ (for both).}$$

Bhatia-Davis

$$\mathrm{Var}[X] \le \big(M - \mathrm{E}[X]\big)\big(\mathrm{E}[X] - m\big), \quad \text{where } X \in [m, M].$$

Samuelson

$$\mu - \sigma\sqrt{n-1} \le x_i \le \mu + \sigma\sqrt{n-1}, \quad \text{for } i = 1, \dots, n.$$

Where $\mu = \sum x_i/n$, $\sigma^2 = \sum (x_i - \mu)^2/n$.

Vysochanskij-Petunin-Gauss

$$\Pr\big[\,\big|X - \mathrm{E}[X]\big| \ge \lambda\sigma\,\big] \le \frac{4}{9\lambda^2}, \quad \text{if } \lambda \ge \sqrt{\frac{8}{3}},$$

$$\Pr\big[\,\big|X - m\big| \ge \varepsilon\,\big] \le \frac{4\tau^2}{9\varepsilon^2}, \quad \text{if } \varepsilon \ge \frac{2\tau}{\sqrt{3}},$$

$$\Pr\big[\,\big|X - m\big| \ge \varepsilon\,\big] \le 1 - \frac{\varepsilon}{\sqrt{3}\tau}, \quad \text{if } \varepsilon \le \frac{2\tau}{\sqrt{3}}.$$

Where X is a unimodal random variable with mode m,
$\sigma^2 = \mathrm{Var}[X] < \infty$, $\tau^2 = \mathrm{Var}[X] + (\mathrm{E}[X] - m)^2 = \mathrm{E}[(X - m)^2]$.

Kolmogorov

$$\Pr\big[\max_k |S_k| \ge \varepsilon\big] \le \frac{1}{\varepsilon^2}\mathrm{Var}[S_n] = \frac{1}{\varepsilon^2}\sum_i \mathrm{Var}[X_i],$$

where $X_1, \dots, X_n$ are independent random variables, $\mathrm{E}[X_i] = 0$, $\mathrm{Var}[X_i] < \infty$ for all i, $S_k = \sum_{i=1}^{k} X_i$ and $\varepsilon > 0$.

Etemadi

$$\Pr\big[\max_{1 \le k \le n} |S_k| \ge 3\alpha\big] \le 3 \max_{1 \le k \le n}\big(\Pr\big[\,|S_k| \ge \alpha\big]\big),$$

where X_i are independent random variables, $S_k = \sum_{i=1}^{k} X_i$, $\alpha \ge 0$.

Bennett

$$\Pr\Big[\sum_{i=1}^{n} X_i \ge \varepsilon\,\Big] \le \exp\left(-\frac{n\sigma^2}{M^2}\,\theta\big(\frac{M\varepsilon}{n\sigma^2}\big)\right), \quad \text{where } X_i \text{ independent,}$$

$\mathrm{E}[X_i] = 0$, $\sigma^2 = \frac{1}{n}\sum \mathrm{Var}[X_i]$, $|X_i| \le M$ (w. probab. 1), $\varepsilon \ge 0$,
$\theta(u) = (1+u)\log(1+u) - u$.

Doob

$$\Pr\big[\max_{1 \le k \le n} |X_k| \ge \varepsilon\big] \le \frac{\mathrm{E}\big[|X_n|\big]}{\varepsilon}, \quad \text{for martingale } (X_k) \text{ and } \varepsilon > 0.$$

Bernstein

$$\Pr\Big[\sum_{i=1}^{n} X_i \ge \varepsilon\,\Big] \le \exp\left(\frac{-\varepsilon^2}{2(n\sigma^2 + M\varepsilon/3)}\right), \quad \text{for } X_i \text{ independent,}$$

$\mathrm{E}[X_i] = 0$, $|X_i| < M$ (w. prob. 1) for all i, $\sigma^2 = \frac{1}{n}\sum \mathrm{Var}[X_i]$, $\varepsilon \ge 0$.

Chernoff

$$\Pr\big[\,X \ge (1+\delta)\mu\,\big] \le \left(\frac{e^{\delta}}{(1+\delta)^{(1+\delta)}}\right)^{\mu} \le \exp\left(\frac{-\mu\delta^2}{\min\{2+\delta, 3\}}\right),$$

where X_i independently drawn from {0,1}, $X = \sum X_i$, $\mu = \mathrm{E}[X]$, $\delta \ge 0$.

$$\Pr\big[\,X \le (1-\delta)\mu\,\big] \le \left(\frac{e^{-\delta}}{(1-\delta)^{(1-\delta)}}\right)^{\mu} \le \exp\left(\frac{-\mu\delta^2}{2}\right), \quad \text{for } \delta \in [0,1).$$

Simpler (weaker) form: $\Pr\big[\,X \ge R\,\big] \le 2^{-R}$, for $R \ge 2e\,\mu$ ($\approx 5.44\mu$).

Hoeffding

$$\Pr\big[\,\big|X - \mathrm{E}[X]\big| \ge \delta\,\big] \le 2\exp\left(\frac{-2\delta^2}{\sum_{i=1}^{n}(b_i - a_i)^2}\right), \quad \text{for } X_i \text{ indep.}$$

$X_i \in [a_i, b_i]$ (w. prob. 1), $X = \sum X_i$, $\delta \ge 0$.

A related lemma, assuming $\mathrm{E}[X] = 0$, $X \in [a, b]$ (w. prob. 1) and $\lambda \in \mathbb{R}$:

$$\mathrm{E}\big[e^{\lambda X}\big] \le \exp\left(\frac{\lambda^2(b-a)^2}{8}\right)$$

Azuma

$$\Pr\big[\,\big|X_n - X_0\big| \ge \delta\,\big] \le 2\exp\left(\frac{-\delta^2}{2\sum_{i=1}^{n} {c_i}^2}\right), \text{ for martingale } (X_k) \text{ s.t.}$$

$\big|X_i - X_{i-1}\big| < c_i$ (w. probability 1), for $i = 1, \dots, n$, $\delta \ge 0$.

Efron-Stein

$$\mathrm{Var}[Z] \le \frac{1}{2}\,\mathrm{E}\left[\sum_{i=1}^{n}\big(Z - Z^{(i)}\big)^2\right], \quad \text{for } X_i, X_i{}' \in \mathcal{X} \text{ independent,}$$

$f : \mathcal{X}^n \to \mathbb{R}$, $Z = f(X_1, \dots, X_n)$, $Z^{(i)} = f(X_1, \dots, X_i{}', \dots, X_n)$.

McDiarmid

$$\Pr\big[\,\big|Z - \mathrm{E}[Z]\big| \ge \delta\,\big] \le 2\exp\left(\frac{-2\delta^2}{\sum_{i=1}^{n} c_i^2}\right), \quad \text{for } X_i, X_i{}' \in \mathcal{X} \text{ indep.,}$$

Z, $Z^{(i)}$ as before, s.t. $\big|Z - Z^{(i)}\big| \le c_i$ for all i, and $\delta \ge 0$.

Janson

$$M \le \Pr\big[\textit{no } B_i \textit{ occurs}\big] \le M\exp\left(\frac{\Delta}{2 - 2\varepsilon}\right), \quad \text{where } \varepsilon \ge \Pr[B_i] \text{ for all } i,$$

$$M = \prod(1 - \Pr[B_i]), \ \Delta = \sum_{i \ne j}\Pr[B_i \wedge B_j], \text{ for } B_i \text{ and } B_j \text{ dependent.}$$

Erdős

$$\sum_{1 \le j < k \le n}\frac{1}{x_k - x_j} \ge \frac{1}{8}n^2\log n, \quad \text{where } -1 \le x_1 \le \cdots \le x_n \le 1.$$

Kraft

$$\sum_{i=1}^{N} 2^{-c(i)} \le 1, \quad \text{where } N \text{ is the number of leaves in a binary tree,}$$

and $c(i)$ is the depth of a leaf i.

References and latest version of this file:
http://www.Lkozma.net/inequalities_cheat_sheet