Sage Quick Reference: Abstract Algebra

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latest version: http://wiki.sagemath.org/quickref

Based on work by P. Jipsen, W. Stein, R. Beezer

Basic Help

com⟨tab⟩ complete *command*
a.⟨tab⟩ all methods for object a
<command>? for summary and examples
<command>?? for complete source code
foo? list all commands containing *foo*
_ underscore gives the previous output
www.sagemath.org/doc/reference online reference
www.sagemath.org/doc/tutorial online tutorial
load foo.sage load commands from the file foo.sage
attach foo.sage
loads changes to foo.sage automatically

Lists

`L = [2,17,3,17]` an ordered list
`L[i]` the ith element of L
Note: lists begin with the 0th element
`L.append(x)` adds x to L
`L.remove(x)` removes x from L
`L[i:j]` the i-th through $(j-1)$-th element of L
`range(a)` list of integers from 0 to $a-1$
`range(a,b)` list of integers from a to $b-1$
`[a..b]` list of integers from a to b
`range(a,b,c)`
every c-th integer starting at a and less than b
`len(L)` length of L
`M = [i^2 for i in range(13)]`
list of squares of integers 0 through 12
`N = [i^2 for i in range(13) if is_prime(i)]`
list of squares of prime integers between 0 and 12
`M + N` the concatenation of lists M and N
`sorted(L)` a sorted version of L (L is not changed)
`L.sort()` sorts L (L is changed)
`set(L)` an unordered list of unique elements

Programming Examples

Print the squares of the integers $0, \ldots, 14$:

```
for i in range(15):
    print i^2
```

Print the squares of those integers in $\{0, \ldots, 14\}$ that are relatively prime to 15:

```
for i in range(13):
    if gcd(i,15)==1:
        print i^2
```

Preliminary Operations

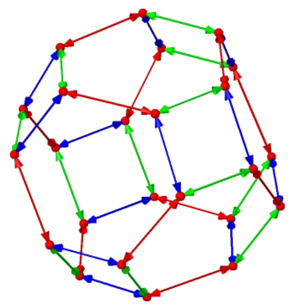
`a = 3; b = 14`
`gcd(a,b)` greatest common divisor a, b
`xgcd(a,b)`
triple (d, s, t) where $d = sa + tb$ and $d = \gcd(a, b)$
`next_prime(a)` next prime after a
`previous_prime(a)` prime before a
`prime_range(a,b)` primes p such that $a \leq p < b$
`is_prime(a)` is a prime?
`b % a` the remainder of b upon division by a
`a.divides(b)` does a divide b?

Group Constructions

Permutation multiplication is left-to-right.
`G = PermutationGroup([[(1,2,3),(4,5)],[(3,4)]])`
perm. group with generators $(1,2,3)(4,5)$ and $(3,4)$
`G = PermutationGroup(["(1,2,3)(4,5)","(3,4)"])`
alternative syntax for defining a permutation group
`S = SymmetricGroup(4)` the symmetric group, S_4
`A = AlternatingGroup(4)` alternating group, A_4
`D = DihedralGroup(5)` dihedral group of order 10
`Ab = AbelianGroup([0,2,6])` the group $\mathbb{Z} \times \mathbb{Z}_2 \times \mathbb{Z}_6$
`Ab.0, Ab.1, Ab.2` the generators of Ab
`a,b,c = Ab.gens()`
shorthand for a = Ab.0; b = Ab.1; c = Ab.2
`C = CyclicPermutationGroup(5)`
`Integers(8)` the group $\mathbb{Z}_8$
`GL(3,QQ)` general linear group of 3×3 matrices
`m = matrix(QQ,[[1,2],[3,4]])`
`n = matrix(QQ,[[0,1],[1,0]])`
`MatrixGroup([m,n])`
the (infinite) matrix group with generators m and n
`u = S([(1,2),(3,4)]);  v = S((2,3,4))` elements of S
`S.subgroup([u,v])`
the subgroup of S generated by u and v
`S.quotient(A)` the quotient group S/A
`A.cartesian_product(D)` the group A×D
`A.intersection(D)` the intersection of groups A and D
`D.conjugate(v)` the group $\mathtt{v}^{-1}\mathtt{Dv}$
`S.sylow_subgroup(2)` a Sylow 2-subgroup of S
`D.center()` the center of D
`S.centralizer(u)` the centralizer of x in S
`S.centralizer(D)` the centralizer of D in S
`S.normalizer(u)` the normalizer of x in S
`S.normalizer(D)` the normalizer of D in S
`S.stabilizer(3)` subgroup of S fixing 3

Group Operations

`S = SymmetricGroup(4); A = AlternatingGroup(4)`
`S.order()` the number of elements of S
`S.gens()` generators of S
`S.list()` the elements of S
`S.random_element()` a random element of S
`u*v` the product of elements u and v of S
`v^(-1)*u^3*v` the element $\mathtt{v}^{-1}\mathtt{u}^3\mathtt{v}$ of S
`u.order()` the order of u
`S.subgroups()` the subgroups of S
`S.normal_subgroups()` the normal subgroups of S
`A.cayley_table()` the multiplication table for A
`u in S` is u an element of S?
`u.word_problem(S.gens())`
write u as a product of the generators of S
`A.is_abelian()` is A abelian?
`A.is_cyclic()` is A cyclic?
`A.is_simple()` is A simple?
`A.is_transitive()` is A transitive?
`A.is_subgroup(S)` is A a subgroup of S?
`A.is_normal(S)` is A a normal subgroup of S?
`S.cosets(A)` the right cosets of A in S
`S.cosets(A,'left')` the left cosets of A in S
`g = S.cayley_graph()` Cayley graph of S
`g.show3d(color_by_label=True, edge_size=0.01, vertex_size=0.03)` see below:



Ring and Field Constructions

`ZZ` integral domain of integers, $\mathbb{Z}$

`Integers(7)` ring of integers mod 7, $\mathbb{Z}_7$

`QQ` field of rational numbers, $\mathbb{Q}$

`RR` field of real numbers, $\mathbb{R}$

`CC` field of complex numbers, $\mathbb{C}$

`RDF` real double field, inexact

`CDF` complex double field, inexact

`RR` 53-bit reals, inexact, not same as `RDF`

`RealField(400)` 400-bit reals, inexact

`ComplexField(400)` complexes, too

`ZZ[I]` the ring of Gaussian integers

`QuadraticField(7)` the quadratic field, $\mathbb{Q}(\sqrt{7}\,)$

`CyclotomicField(7)`
smallest field containing $\mathbb{Q}$ and the zeros of $x^7 - 1$

`AA, QQbar` field of algebraic numbers, $\overline{\mathbb{Q}}$

`FiniteField(7)` the field $\mathbb{Z}_7$

`F.<a> = FiniteField(7^3)`
finite field in a of size 7^3, $\mathrm{GF}(7^3)$

`SR` ring of symbolic expressions

`M.<a>=QQ[sqrt(3)]` the field $\mathbb{Q}[\sqrt{3}]$, with $a = \sqrt{3}$.

`A.<a,b>=QQ[sqrt(3),sqrt(5)]`
the field $\mathbb{Q}[\sqrt{3}, \sqrt{5}]$ with $a = \sqrt{3}$ and $b = \sqrt{5}$.

`z = polygen(QQ,'z'); K = NumberField(x^2 - 2,'s')`
the number field in s with defining polynomial $x^2 - 2$

`s = K.0` set `s` equal to the generator of `K`

`D = ZZ[sqrt(3)]`

`D.fraction_field()`
field of fractions for the integral domain `D`

Ring Operations

Note: Operations may depend on the ring

`A = ZZ[I]; D = ZZ[sqrt(3)]` some rings

`A.is_ring()` is A a ring?

`A.is_field()` is A a field?

`A.is_commutative()` is A commutative?

`A.is_integral_domain()`
`True` is A an integral domain?

`A.is_finite()` is A is finite?

`A.is_subring(D)` is A a subring of D?

`A.order()` the number of elements of A

`A.characteristic()` the characteristic of A

`A.zero()` the additive identity of A

`A.one()` the multiplicative identity of A

`A.is_exact()`
`False` if `A` uses a floating point representation

`a, b = D.gens(); r = a + b`

`r.parent()` the parent ring of r (in this case, `D`)

`r.is_unit()` is r a unit?

Polynomials

`R.<x> = ZZ[ ]` `R` is the polynomial ring $\mathbb{Z}[x]$

`R.<x> = QQ[ ]; R = PolynomialRing(QQ,'x'); R = QQ['x']`
`R` is the polynomial ring $\mathbb{Q}[x]$

`S.<z> = Integers(8)[ ]` `S` is the polynomial ring $\mathbb{Z}_8[z]$

`S.<s, t> = QQ[ ]` `S` is the polynomial ring $\mathbb{Q}[s,t]$

`p = 4*x^3 + 8*x^2 - 20*x - 24`
a polynomial in `R` ($= \mathbb{Q}[x]$)

`p.is_irreducible()` is p irreducible over $\mathbb{Q}[x]$?

`q = p.factor()` factor p

`q.expand()` expand `q`

`p.subs(x=3)` evaluates p at $x = 3$

`R.ideal(p)` the ideal in R generated by p

`R.cyclotomic_polynomial(7)`
the cyclotomic polynomial $x^6+x^5+x^4+x^3+x^2+x+1$

`q = x^2-1`

`p.divides(q)` does p divide q?

`p.quo_rem(q)`
the quotient and remainder of `p` upon division by `q`

`gcd(p, q)` the greatest common divisor of p and q

`p.xgcd(q)` the extended gcd of p and q

`I = S.ideal([s*t+2,s^3-t^2])`
the ideal $(st+2, s^3-t^2)$ in `S` ($= \mathbb{Q}[s,t]$)

`S.quotient(I)` the quotient ring, S/I

Field Operations

`A.<a,b>=QQ[sqrt(3),sqrt(5)]`

`C.<c> = A.absolute_field()`
"flattens" a relative field extension

`A.relative_degree()`
the degree of the relative extension field

`A.absolute_degree()`
the degree of the absolute extension

r = a + b; `r.minpoly()`
the minimal polynomial of the field element `r`

`C.is_galois()` is `C` a Galois extension of Q?